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CONTENTS

March, 1960

Papers

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Proceedings of the American Society of Civil Engineers

ITALIAN ARCH DAM DESIGN AND MODEL CONFIRMATION

By Guido Oberti,[1] F. ASCE and M. IABSE

FOREWORD

This paper was the basis for an oral presentation at the Joint ASCE-IABSE Meeting at the New York Convention, October 1958. All Joint Meeting papers published in Proceedings or in Civil Engineering will be reprinted in one volume.

SYNOPSIS

This paper deals with the interdependence of art, experience, and science in the design of large dams. The use of improved analytical methods and large-scale model tests in determining the static behavior and stability of dams is presented, and the use of models in the preliminary design stage is discussed.

The statical characteristics of the principal arched dams in Italy are outlined, and the results of model tests of these dams, made at the ISMES research laboratory, are given.

Note.—Discussion open until August 1, 1960. Separate Discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 3, March, 1960.

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PRELIMINARY CONSIDERATIONS

A rational dam design is of fundamental technical and economic importance in the execution of hydraulic projects.

The basic concept of arch dam design is that the structure utilizes both the physical properties of the concrete and the shape of the structure to resist the hydrostatic pressure imposed on it. An arch dam can be conceived as a monolithic double-curvature shell which resists and distributes the load along its perimeter to the rock abutments and the bottom of the dam site. Hence, the gravity load effect usually becomes of secondary importance to the effect of the shape of the structure in resisting the hydrostatic pressure most efficiently.

Nature offers admirable examples of structures that resist pressure primarily because of their shape, such as conchs, eggshells, and some flower calyxes. On the other hand, various man-made structures, such as hulls of certain boats, large amphorae, and glass or ceramic vases, provide wonderful and artistic examples of structures which also resist hydrostatic load mainly on account of their shape.

TRENDS IN MODERN CONCRETE DAM DESIGN

The basic structural types which have been used in building concrete dams are the gravity type, in which the resistance to hydrostatic pressure is dependent solely on the weight of the masonry, and in which the stresses are essentially combined compression and bending; and the arch type, in which resistance is primarily due to the shape of the structure, and in which the structure, acting chiefly as a membrane, is subjected to pure axial compression.

The earliest arch dams were of single curvature, in which the arch resistance was thought of and calculated by assuming the structure as subdivided by horizontal planes into numerous elementary arch rings, individually and independently resisting the hydrostatic pressure.

In more recent years the design concept for arch dams has changed from single-curvature structures to double-curvature, dome-like structures. This change began in Italy some 20 yr ago, much through this writer's initiative. In double-curvature arch dams, the basic resistance is truly a compressive membrane, so that the hydrostatic pressure is resisted at every point by a framework of variously inclined arches which are supported by the nearest abutments or by the crest arch. The Osiglietta Dam, shown in Fig. 1, was the first striking example of this kind of dam. This dam, constructed about 1938, was designed by Fabio Niccolai, with the writer as a consultant, and constructed by Mario Scalabrini.

The topography of the dam site determined, even though incorrectly, the range of application of the two extreme structural types mentioned previously. Gravity dams were used where the width of the valley greatly exceeded the height of the dam, whereas arch dams were given consideration where the width of the dam site was not greater than twice the expected maximum height of the dam. Shortly after 1940, a broad valley of the Piave River was to be dammed a short distance downstream from Pieve di Cadore. According to design criteria prevailing at that time, the dam should have been of the gravity type, since the width of the valley was about six times the mean height of the dam. However, Carlo Semenza, F. ASCE, proposed an arch dam as the best solution of the problem. Thus began the study of arch-gravity dams, an intermediate type of dam in which the hydrostatic pressure is resisted partly by arch action

and partly by gravity action. Simultaneously, other dams of this type were constructed abroad, of which Rossens Dam in Switzerland and Boulder Dam in the United States might be mentioned. Since that time, the use of dams of this type has spread widely in Italy, reaching length-to-height ratios even more remarkable than that of the Piave Dam.

The present trend in Italy is towards dome-like, double-curvature structures, even for dam sites with quite large width-to-height ratios. The double curvature makes it possible to produce structures which are extremely thin compared with the gravity type. A comparison of the cross-sections of some of the double-curvature dams designed in Italy, shown in Fig. 2, illustrates clearly the thinness of such structures.

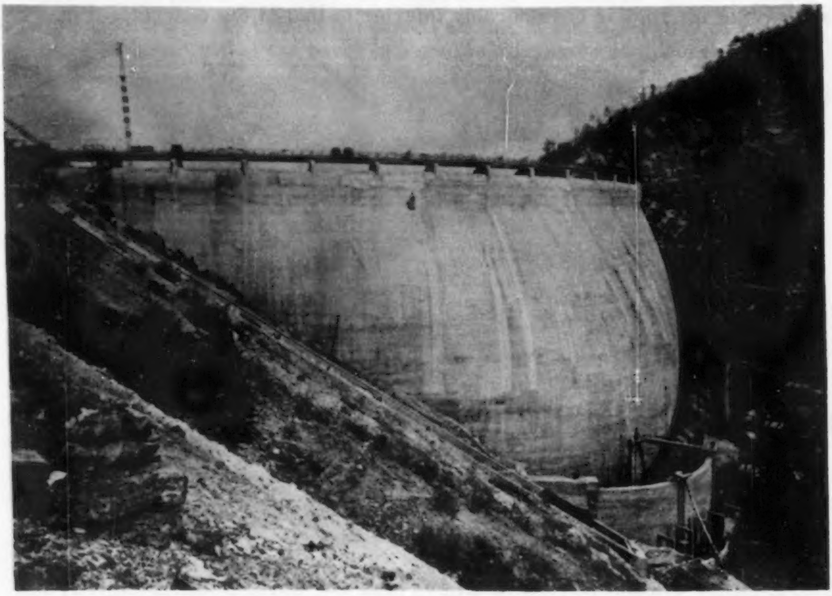

FIG. 1.—OSIGLIETTA DAM.

The writer's personal criterion is to tend toward designing structures in which the compressive stress is distributed throughout the structure as nearly uniformly as possible. The double-curvature structure lends itself particularly well to such distribution. Naturally, model tests are of great assistance in the preliminary design stage for improving the structural form to meet as closely as possible the above criterion. This is the criterion of greatest rationality, and results in the utmost economy. The bi-axial and tri-axial compression induced by membrane action actually improves the strength characteristics of the concrete, as shown by recent tests carried out at the Experimental Institute for Models and Structures (ISMES) at Bergamo, Italy.

In addition to the influence of the length-to-height ratio of the dam site in directing a designer's attention toward one of the fundamental types of dams, the shape of the dam site is also of fundamental importance. Dam sites are

only rarely regular and symmetrical; most often the valley to be dammed is decidedly unsymmetrical. For such cases there are two wholly distinct design solutions. The first solution is to make the dam site symmetrical by excavation, such as was done for the 450 ft high Lumiei arch dam, built in 1943, in order to get a symmetrical structure which was more rational from the standpoints of calculation and construction. The other solution is to keep the dam asymmetrical in conformity with the shape of the valley and to try to attain symmetry of stress by proper variation of the thickness of the structure. This was the solution utilized in designing the arch-gravity dams of Forte Buso and Ponte Racli.

Lastly, the physical characteristics of the foundation rock may make it necessary to modify the basic criterion of uniform stress distribution. If the strength of the rock is considerably inferior to that of the concrete, it may be

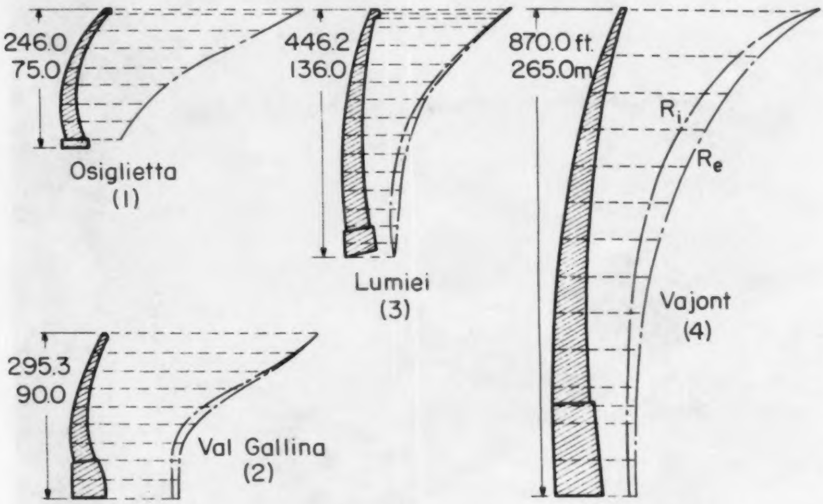

FIG. 2.—MAIN CROSS-SECTIONAL ELEVATION OF SOME DOUBLE-CURVATURE DAMS IN ITALY.

necessary to reduce the maximum compression at the abutments considerably below that of the concrete. The designer may choose one of two ways to accomplish this result: by resorting to ample abutment plinths or saddles, or by gradually increasing the thickness of the cross-section of the dam toward the abutments. This was done for the Mucone Dam, which spans an almost symmetrical site, but for which the abutment stresses had to be reduced considerably to the low strength conditions of the Sila granite foundation rock.

Sometimes a dam site is structurally asymmetrical, even though the topography is symmetrical, because of different strength characteristics of the rock on the two abutments. The Val Gallina double-curvature arch dam, shown in Fig. 3, and the Beauregard arch-gravity dam are examples of this condition. For these cases the writer proposed that the stresses along the abutments of both dams be made unsymmetrical, conforming to the unequal strength of the foundations.

Finally, the design of the dam may be profoundly influenced by construction conditions. For example, if a dam is to be constructed in successive stages, a single-curvature structure may be advantageous compared to a double-curvature structure, because the former has no parts that are out-of-plumb, and is therefore easier to construct in successive stages. The large arch-gravity dams under construction at Cancano and at Frera in the Italian Alps are examples of the influence of construction methods on design.

DOUBLE-CURVATURE DAM STUDY AND DESIGN CRITERIA

In a single-curvature arch dam, considerable secondary tensile stresses are found along the "cantilevers." These act as rectilinear, nearly vertical beams supported by the elastic horizontal arches at the upper part of the dam but more or less fixed at the base. In a double-curvature dam, the cantilevers

FIG. 3.—VAL GALLINA DAM.

are replaced by nearly vertical arches which extend from the base to the crest of the dam, and thus offer a better support for the horizontal arches in the main portion of the dam.

In order to avoid local stress concentrations, it is desirable to have the utmost possible continuity of the variation of both the curvature and the thickness of the dam. In other words, the radii and thicknesses of the horizontal and vertical cross-sections of the dome-like structure should be increased gradually toward the foundations in conformity with continuous, analytically-defined functions, as shown in Fig. 4.

The dam should be provided with a perimeter joint and intermediate radial joints. The importance of such joints now is well known. They constitute discontinuities that prevent the transmission of possible tensile stresses throughout the structure, since they localize the effect of the contraction of the concrete caused by thermal changes and drying shrinkage, and thus avoid cracking.

The favorable static behavior of double-curvature dams compared with those of single curvature has been definitely established by the ultimate load model tests as essentially due to the shape of the structure, which gives it great inherent stiffness. Whereas in a single-curvature dam it is necessary that the

geometric axes of the arches correspond as closely as possible to the pressure curves to minimize bending stresses, in dome-like dams this is of much less importance in view of the load taken by the cantilevers which act as vertical arches. In other words, by designing the horizontal arches as funicular polygons of the radial hydrostatic pressure, they support most of the external load and leave only a small part of the load to be carried by the arched vertical members, which are also laid out as funicular polygons. Consequently, the bending moments along such cantilevers are also very much reduced, and the tensile stresses are negligible.

The analysis of arch dams is in a continuing state of development. The analysis of single-curvature arch dams has benefited greatly by the work of Westergaard, Smith, Tölke, and Italian investigators. However, for double-curvature

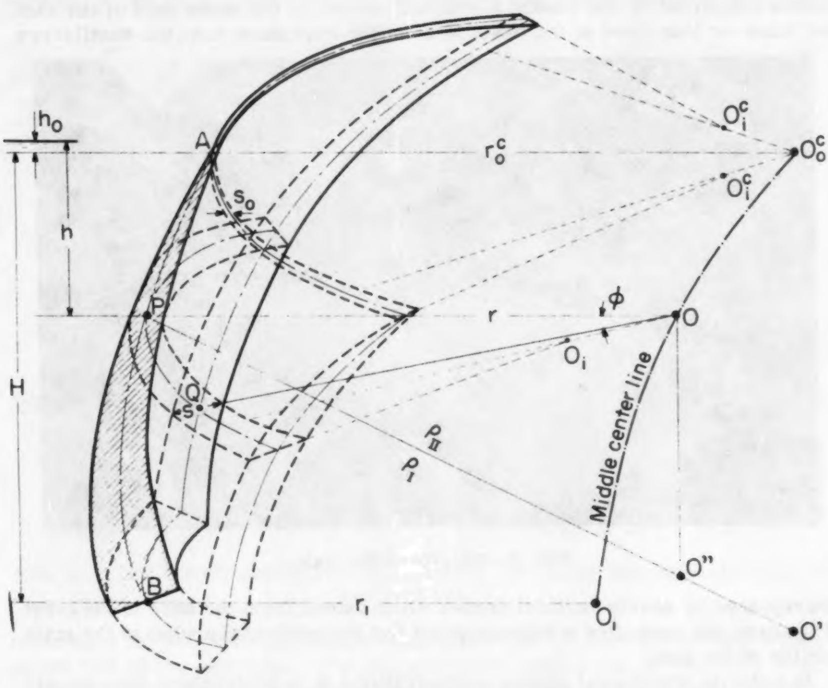

FIG. 4.—SKETCH OF A DOUBLE-CURVATURE DAM AND MAIN CROSS-SECTIONAL ELEVATION.

arch dams, there is no method of calculation that will give satisfactory solutions based on acceptable initial assumptions without extremely burdensome labor, mainly because the differential equations encountered in the analysis of such structures are necessarily of the partial differential type. Thus, they are not reduceable, by means of approximate assumptions, to ordinary differential equations, particularly if the central angle is large, and because the boundary conditions, especially those of the foundation rock, can hardly be ascertained by analytical means. It is therefore better to rely on model tests, which have systematically corroborated, up to ultimate load, the anticipated resistance contributed by the arched shape of the cantilevers.

It would be logical, therefore, to permit higher allowable stresses in horizontal arches when calculated to resist the hydrostatic pressure as individual horizontal arch rings unaided by the cantilevers. For prudence's sake, however, in Italy the verification of the stability of the dam is normally carried out to conform with the existing Official Specifications, which excludes consideration of any contribution by the vertical members during the initial calculation of the arches, but allows comparatively high maximum stresses (850 psi to 1000 psi) as these are more theoretical then actual. The stability check is then continued by introducing the stiffening effect of the cantilevers, in the high central portion of the dam. As a first approximation, the cantilevers are necessarily assumed as members of a thin rotational shell. This calculation furnishes an idea of the order of magnitude of the mean compressive stresses resulting from the second curvature, which are beneficial to the stability of the structure.

In the analysis of the independent horizontal arches, consideration is also given to the thermal and shrinkage actions, whereas the analysis of the stiffening influence of the cantilevers takes into account only the effect of dead load and the vertical component of the hydrostatic pressure.

In the elastic analysis of the isolated arches, which is always of basic importance because of the predominating influence of such members in the total resistance of the structure, the assumption is made that they are continuous and elastically fixed at the abutments. The arches are usually designed of varying thickness from a minimum at the crown increasing ever more rapidly to a maximum at the abutments. The cantilevers in double-curvature structures are arched so as to cooperate most effectively with the horizontal arches to support both the hydrostatic head and the dead load. Compatibly with other design requirements, the cantilevers are shaped to follow the vertical-plane funicular polygon of the hydrostatic pressure in the central portion of the dam, gradually joining the abutments and the bottom.

The central portion of the dam is the only part that can be submitted to analysis, as model tests show that only here is the trend of the isostatic lines on the two faces close to the structure's meridians (cantilevers) and parallels (arches). Therefore, in that area it is sufficiently accurate to neglect the torsional stresses and the shearing stresses not only in the arches but also in the plane tangent to the structure (membrane-shearing stresses).

The double-curvature arch dams designed by us may be considered as membranes of a comparatively moderate and conveniently varying thickness. The mean surface of these membranes may be thought of as generated by a moving arc of a circumference with vertex A, center 0 and variable radius r (Fig. 4). As the arc moves along, vertex A describes an arched directrix (main meridian AB), while radius A0 stays horizontal and in the vertical plane which contains the directrix.

A dam element having a thickness s and located between two infinitely close arches and two cantilevers may be considered as a revolution-surface element in a state of equilibrium under the action of the hydrostatic pressure ($p_e = \gamma h$), the dead load ($q = \gamma_c s$), and the resultants of the forces acting on the four boundary faces. Referring to the mean surface of the element, its two main curvature radii are respectively ρ_I for the meridian and $\rho_{II} = r/\sin \phi$ for the arcs normal to the meridian, where r is the radius of the corresponding parallel or horizontal arc. As shown in Fig. 5, the forces acting on the mean surface of the element are

$$P = p\ r\ d\theta\ \rho_I\ d\phi = p\ dx\ ds$$

$$Q = q\ dx\ ds$$

Lastly, the forces per unit length of the face on which they act will be limited, for the assumed symmetry, to:

N_a, the thrust on the arch element
N_c, the thrust on the cantilever element
M_a, the moment on the arch element
M_c, the moment on the cantilever element
T_c, the shear on the cantilever element

The equations defining the equilibrium conditions of the element reduce to:

$$p' = \frac{N_a}{\rho_{II}} + \frac{N_c}{\rho_I} + \frac{1}{r}\ \frac{d}{ds}(T_c\ r) \qquad (1a)$$

$$q' = \frac{T_c}{\rho_I} - \frac{1}{r}\ \frac{d}{ds}(N_c\ r) + \frac{N_a}{\rho_{II}}\ \cot\ \phi \qquad (1b)$$

and

$$T_c = \frac{1}{r}\ \frac{d}{ds}(M_c\ r) - \frac{M_a}{\rho_{II}}\ \cot\ \phi \qquad (1c)$$

where $p' = p + q\cos\phi$ (intensity of normal load), and $q' = q\sin\phi$ (intensity of tangential load). The problem thus remains inherently statically indeterminate despite the assumption of complete symmetry.

In the preliminary design stage it is still possible to assume a predominantly membrane-like behavior. This approximation becomes increasingly valid the more slender the dam. In such a case it is permissible to neglect the flexural strength (that is M_a, M_c, and hence T_c) so that Eqs. 1 reduce to two equations, namely:

$$p' = \frac{N_c}{\rho_I} + \frac{N_a}{\rho_{II}} \qquad (2a)$$

$$N_a = \frac{1}{\cos\phi}\left[\frac{d}{ds}(N_c\ r) - q'\ r\right] \qquad (2b)$$

The problem thus becomes statically determined.

Eq. 2a is the classical dome formula. It furnishes, at the point under consideration, the division of the normal load p' into two parts:

$$p_c' = \frac{N_c}{\rho_I} \qquad (3a)$$

supported by the cantilever, and

$$p_a' = \frac{N_a}{\rho_{II}} \qquad (3b)$$

supported by the arch.

Having fixed the crest arch conditions, Eq. 2b makes it possible to obtain the values of the stresses N_c progressively downward along the cantilever, and using Eq. 2a, the values of N_a along the arches. It is thus essential to analyze the equilibrium of the structure's highest central cantilevers bounded by two radial planes separated by a unit length. The central cantilever is divided into a number of elementary blocks, each having a length Δs and bounded by perpendicular planes. The known hydrostatic pressure and dead load forces acting on one of these parts are balanced by the horizontal thrusts opposed by the arch, of which each block may be considered a portion, and by the internal forces acting along the cantilever. The internal forces may be assumed as centered and normal to the cross-sections bounding the block itself. Starting from the

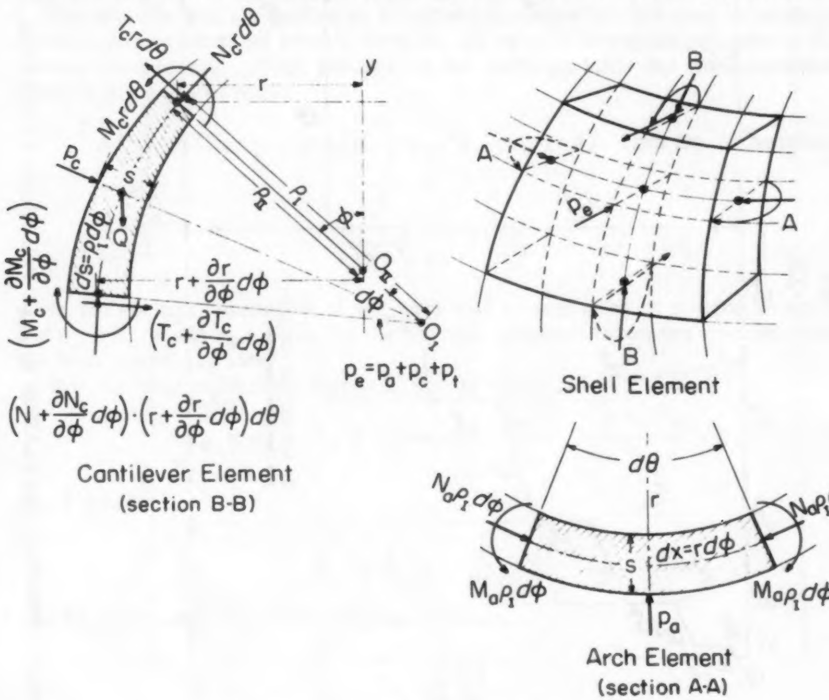

FIG. 5.—DOUBLE-CURVATURE DAM ELEMENT.

uppermost block, it is then possible to construct graphically, as shown in Fig. 6, the successive equilibrium polygons and evaluate both the pressure N_c along the cantilever and the resultant S of the sustaining arch thrust N_a. Once these forces are known, the transition to the corresponding stresses is direct and immediate. There appears clearly along the cantilever a flow of mean compressive stress that gradually increases from the crest to the base. The method of calculation by isolating the members also shows clearly that the horizontal arches in the central and lower portions of the dam become partly relieved, whereas in the upper region they become more stressed, as can be seen in Fig. 6 by comparing forces H and S.

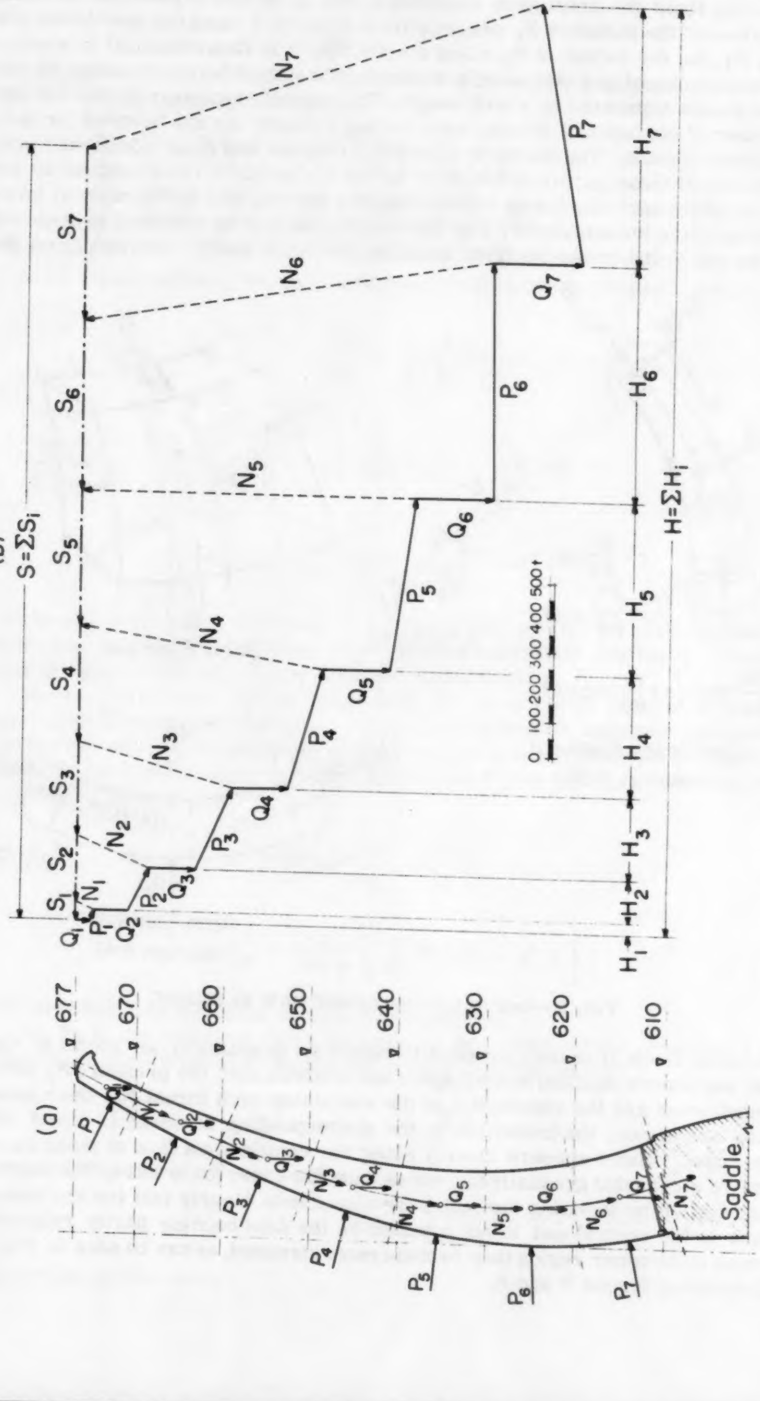

FIG. 6.—GRAPHIC CALCULATION METHOD FOR CROWN CANTILEVER.

It may be concluded that the effects produced by double-curvature are:

1. To relieve those arches which, when analyzed as isolated horizontal members, appear to be the most highly stressed, and to increase the compression of the upper arches, which from the same analysis are but little stressed.
2. To facilitate the formation of mean compressive stresses along the vertical cantilevers.

The analysis confirms that, in the elastic stage, the basic resisting elements are the arches, whereas the cantilevers offer a comparatively limited contribution owing to the curvature ratios of the structure.

The relation stating the law of thickness variation for the case of uniform strength can be obtained readily from Eq. 2a by converting the equation to dimensionless terms. First assume as the main variable the dimensionless quantity $\xi = h/H$ and make

$\rho_I' = \rho_I/H$, $\rho'_{II} = \rho_{II}/H$, $\zeta = s/s_o = \zeta(\xi, \theta)$. Then Eq. 2a becomes

$$\frac{\gamma H^2}{s_o \zeta} \xi = \frac{\sigma_c}{\rho_I'} + \frac{\sigma_a}{\rho_{II}'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

when the normal component of the dead load is neglected by putting $p' = \gamma h$, and $\sigma_c = N_c/s$ and $\sigma_a = N_a/s$ for the normal membrane stresses exerted along the main curvature lines.

For the case of uniform strength, that is when

$$\sigma_c = \sigma_a = \sigma_o = \text{constant} = \frac{p_o r_o}{s_o} = \frac{\gamma h_o r_o}{s_o} \quad \ldots\ldots\ldots \quad (4)$$

Eq. 3 becomes

$$\frac{\gamma H^2}{\sigma_o s_o} \frac{\xi}{\zeta(\xi, \theta)} = \frac{1}{\rho_I'} + \frac{1}{\rho_{II}'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

and by introducing the dimensionless constant

$$c = \frac{\gamma H^2}{\sigma_o s_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

and making

$$\frac{1}{\rho_I'} + \frac{1}{\rho_{II}'} = \frac{1}{\rho'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

the law of thickness variation is obtained as

$$\zeta(\xi, \theta) = c\, \rho'\, \xi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

In this relation, ρ' (which is equal to the ratio of the mean radius of curvature of the membrane to H) is geometrically defined by the morphological conditions of the problem under consideration, and is hence a known function of ξ and θ.

EQUATIONS OF GENERAL EQUILIBRIUM

The general behavior of an arch dam may be approximated by combining the "membrane" behavior, which produces only axial and shearing forces, and the "slab" behavior, which contributes bending and twisting moments. Assuming that the straight line connecting the centers of curvature of the horizontal arches is vertical, the ultimate general equilibrium equation of a symmetrical arch element whose hydrostatic pressure, with reference to the central fiber, is p, is as follows:

$$p = \left[\frac{N_a}{r} \sin \phi\right] + \left[-\frac{1}{r}\frac{\partial}{\partial y}(M_a \sin \phi) + \frac{\partial^2}{\partial x^2} M_a\right] + \left[\frac{N_c}{\rho_I}\right] + \left[\frac{1}{r}\frac{\partial^2}{\partial y^2}(M_c\, r_c) - \frac{1}{\rho_I}\frac{\partial}{\partial x} M_c\right]$$

$$+ \left[\frac{1}{r}\frac{\partial}{\partial x} M_t \frac{\partial r}{\partial y} + \frac{\partial^2}{\partial x \partial y} M_t + \frac{1}{r}\frac{\partial^2}{\partial x \partial y}(M_t\, r)\right] \ldots\ldots (9)$$

where the terms between the brackets denote, in the order given, the following pressure parts:

p_{an} = those supported by the horizontal arches by the membrane effect;
p_{am} = those supported by the horizontal arches by the flexural effect;
p_{cn} = those supported by the vertical cantilevers by the membrane effect;
p_{cm} = those supported by the vertical cantilevers by the flexural effect; and
p_t = those supported by oblique members by torsional effects.

Hence:

$$p = (p_{an} + p_{am}) + (p_{cn} + p_{cm}) + p_t = p_a + p_c + p_t \ldots . (10)$$

It is interesting to point out that model test results may be used to determine the distribution of the hydrostatic pressure among the various resisting members into which the dam may be divided, provided that the central part may be likened to a portion of a symmetrical arch sufficiently thin to allow the assumption of linear stresses all along the thickness.

The internal action parameters N_c, M_c, N_a, M_a, (T_c, T_a) T, M_t are then easily deduced from the stresses obtained on the structure's faces by model tests:

$$N_a = s\,\frac{\sigma_m + \sigma_v}{2} \qquad M_a = \frac{s^2}{6}\,\frac{\sigma_m - \sigma_v}{2}$$

$$N_c = s\,\frac{\sigma_m' + \sigma_v'}{2} \qquad M_c = \frac{s^2}{6}\,\frac{\sigma_m' - \sigma_v'}{2}$$

$$T_c = s\,\frac{\tau_m + \tau_v}{2} \qquad M_t = \frac{s^2}{6}\,\frac{\tau_m - \tau_v}{2} \qquad \ldots\ldots . (11)$$

where σ'_m = upstream stresses in the direction of the cantilever, σ'_v = downstream stresses in the direction of the cantilever, σ_m = the upstream stresses in the direction of the arch, σ_v = the downstream stresses in the direction of the arch, τ_m = the upstream tangential stress, and τ_v = the downstream tangential stress.

The calculation of such distribution is to be confined to the central cantilever. The contribution afforded by the oblique members P_t cannot usually be evaluated directly for lack of sufficient test data to permit the determination of the twisting and shearing actions in the model. Such a contribution must therefore be established in an approximate way by difference.

The above principles will be illustrated by actual results of dams tested by us in Italy.

MODEL TESTING

In Italy, structural model testing has developed greatly in the past few years. Following their chronological evolution, the methods used in such testing may be divided into three distinct groups:

The first group deals only with plane elastic problems. In this group the most prominent methods are the photoelastic and influence diagram methods.

The second group concerns three-dimensional elastic problems, making use of strain gages for direct model measurements of local deformations. These methods have been used in the study of numerous structures, especially for dams. These methods use materials for casting the model far different from those of the prototype, provided that they are elastic and conform to Hooke's Law, assuming the continuity of the structure, and approximating the boundary conditions. The model then functions as a clever stress-calculating machine and furnishes results that may be compared usefully with those obtained analytically in the study of the elastic behavior of the structure.

Lastly, having ascertained that some structures, especially concrete structures, do not behave according to theory, but behave better than expected, the third group comprises all those methods which are aimed at attempting to obtain behavior in the model like that of the prototype, rather than confirming calculations. This is decidedly a step toward conformity with nature, and it is being followed by ISMES, where not only the behavior of the material is reproduced closely, but also the shape of the structure and construction and boundary conditions.

Tests are thus performed on the model in two distinct and successive groups. In normal load tests, deformations are determined for values close to the similitude conditions under loads corresponding to the normal operational loads of the structure. It is important to point out that at the first load application there may occur inelastic deformation such as foundation yield, joint adjustment, and partial openings or local plasticity which must be stimulated in the model by means of repeated load cycles in order to reach a normal model performance suitable for deformation measurements. These then can be used to evaluate the deflections, stresses, and static behavior of the prototype during normal service.

In the second test series, ultimate load tests are carried out by a gradual increase in load to the collapse of the model. The overall factor of safety K_s of the prototype can be defined as the ratio between the maximum load actually supported and the load considered as normal.

Ultimate load tests give evidence of the peculiar static resources of the structure, the adjustments due to plastic deformation, and possible cracking, as shown in Fig. 7. These tests make possible an evaluation of the actual structural efficiency of the projected structure.

REQUIREMENTS OF SIMILITUDE

If λ and X indicate, respectively, the similitude ratios of the lengths l and forces f, in passing from the prototype to the model:

$$l/l' = \lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(12a)}$$

and

$$f/f' = X \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(12b)}$$

It then follows that the basic dimensionless ratio relating these two quantities to the fundamental unknown represented by the stress (σ) must remain unvaried.

FIG. 7.—DOWNSTREAM CRACKING AT CANCANO DAM AFTER ULTIMATE-LOAD TESTING.

In other words, the similitude ratio (ζ) between the corresponding stresses must be

$$\zeta = \sigma/\sigma' = X/\lambda^2 \ldots\ldots\ldots\ldots\ldots\ldots \text{(13)}$$

The ratio of the other physical quantities involved in the problem will then have to be of the same value. This ratio, termed the "efficiency ratio" must also govern the mechanical properties of the materials of the models and their foundations. This condition constitutes one of the major difficulties encountered in the testing field.

In the particular case of mass or volume forces acting alone, as dead load or hydrostatic pressure acting on dams, there will be $X = \rho\ \gamma^3$ where ρ indi-

cates the density ratio $\rho = \gamma/\gamma'$. The similitude condition will then be identified with the characteristic expression

$$\zeta = \rho \ \lambda \qquad (14)$$

If materials can be found for which ζ remains unvaried, that is, should the intrinsic curve of the model material be similar to that of the prototype material at constant ratio ζ, and the scale and load ratios meet the above conditions, then similitude will be attained not only within the elastic range but up to the breaking point.

In the particular case of arch dams, ultimate-load model tests are carried to the collapse of the model or to the appearance of first cracks on the upstream face. The safety factor K_s will then be given by the ratio between the maximum final value γ'_m, the final density of loading, and γ', the density of the actual liquid under normal operating conditions. And since by the similitude condition (Eq. 14)

$$\rho = \frac{\gamma}{\gamma'} = \frac{\zeta}{\lambda} \qquad (15)$$

and since the specific gravity of the water acting on the prototype is 1, it will ultimately follow that

$$K_s = \frac{\gamma'_m}{\gamma} = \gamma'_m \ \frac{\zeta}{\lambda} \qquad (16)$$

In the case of gravity or arch-gravity dams in which the gravity effect has an essential stabilizing function, it is necessary to arrange for the increase of this effect in order to maintain it at the correct ratio with respect to the hydrostatic pressure. Having taken such precautions, the ultimate load tests may be carried out. In case a sufficient safety margin is reached without breaking the model, it may be interesting to carry the loads further by increasing only the hydrostatic pressure but not the gravity load so as to ascertain the resistance to extraordinary pressures such as those due to earthquakes, ice, or bomb action.

The model, built similarly to the prototype in a conveniently selected scale ratio λ, may very faithfully reproduce the particular boundary conditions, the various discontinuities of the dam, such as radial and perimetral joints, plugs, saddles and galleries, as well as the deformability of the materials constituting the dam and the foundation rock. In this connection it may be pointed out that it is possible to reproduce the irregularities, fractures, faults and anisotropic characteristics of the bedrock which have been determined by field surveys and which are believed to be capable of influencing the stability of the dam.

In the normal load tests the basic problem is that of evaluating the components of stress throughout the model, or at least in a sufficiently large number of locations usually selected on the upstream and downstream faces of the dam. In view of the general impossibility of direct stress measurement, local deformations are determined by means of strain-gages of very small gage length.

Casting of the model must be done by specialists. It has been our experience that it is best to make a preliminary model in wood or plaster, as shown in Fig. 8, to be used for checking the lines of the drawing, preparing the molds and counter-molds for casting the final model, and executing the drawings for the loading and measuring equipment.

The actual models are made of special materials having a high efficiency ratio ζ, and hence low moduli of elasticity, in order to obtain greater strains at equal stress values and correspondingly low ultimate loads and thus to facilitate the ultimate-load tests. The model material most used at ISMES is a mortar made of cement and Liparian pumice stone. When it is desired to proceed beyond the elastic range, materials must be used which are similar to those

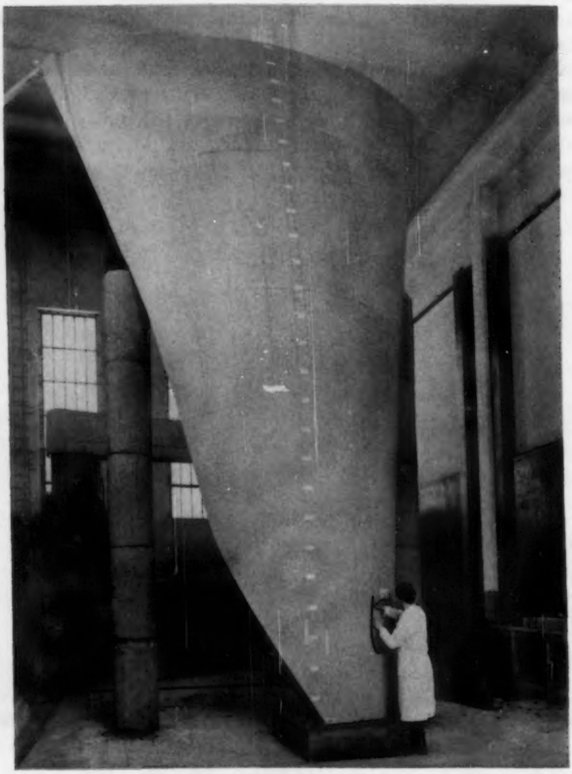

FIG. 8.—PRELIMINARY MODEL OF VAJONT ARCH DAM.

of the prototype for the entire range of stress and strain; the research development in this field is unceasing.

RESULTS OBTAINED IN ITALY FROM ARCH DAM MODEL TESTS

The first tests carried out by the writer at the Milan Polytechnic Institute, concerned the 230 ft high Rocchetta single-curvature dam built in 1937. The model, which was cast in a plaster-celite mix, was tested not only within the elastic range, but also in the ultimate-load range. The investigation led to a favorable conclusion about the static behavior of the dam, and demonstrated

not only the stresses due to the hydrostatic load but, particularly and for the first time in Italy, the great safety resources of arch dams placed on highly rigid foundations.

About this time the idea occurred to replace single-curvature structures by double-curvature dams. Shortly afterwards, the opportunity to use this idea was presented in connection with the Osiglietta Dam, intended to close a very broad mountain gorge, 660 ft wide and 250 ft high, which was quite exceptional in those days for an arch dam. In order to compare the classical single-curvature dam and the double-curvature dam, both types were tested, exclusively within the elastic range, by means of mercury-loaded celluloid models. The tests confirmed the reduction, through the second curvature, of the tensile stresses along the cantilevers, and this proved conclusively the contribution afforded by the cantilevers because of their arched shape.

A double curvature structure was tested again a few years later for the Val Gallina Dam spanning a gorge similar to that of the Osiglietta Dam, but of

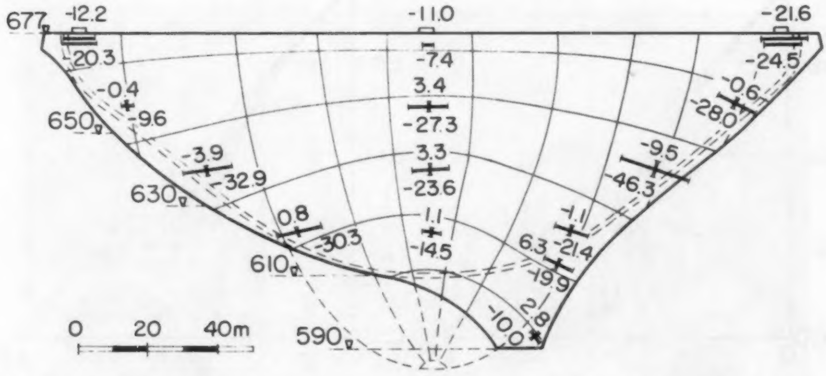

FIG. 9.—PRINCIPAL STRESSES AND ISOSTATIC LINES ON THE DOWNSTREAM FACE OF THE VAL GALLINA DAM.

larger proportions: 820 ft wide and 300 ft high. The rock conditions were also different, as the rock consisted of somewhat deformable fractured limestone, and there was, moreover, a bottom plug closing a somewhat unsymmetrical river bed. Early tests, conducted on a 1:75 scale model, cast in a cement-pumice mortar, furnished the stresses due to the hydrostatic load only, and confirmed the favorable results already obtained in the previous Osiglietta Dam study. As shown in Fig. 9, it can be seen that the characteristic trend of the isostatic lines does not follow at all the traditional vertical-horizontal lattice pattern of cantilevers and arches. This results in a gradual reduction of the axial stresses in the crest arch, proceeding from the crown towards the abutments, and in a corresponding increase in the maximum compressive stresses along the downstream abutments in a zone between about a third and a half the height of the dam.

In a subsequent Val Gallina Dam model, tested at the ISMES Laboratory for the determination of the deformability and non-uniformity effects of the foundation rock, the part of the valley affected by the erection of the dam was reproduced. This model also reproduced the modifications introduced into the project during the construction stage of the dam, such as a special perimeter

joint which was complete at the sides and confined only to the upstream face in the central portion of the dam.

The static behavior of such domes or shells may be visualized by means of diagrams, shown in Fig. 10, which have as a starting point the measurements performed at the central cantilever. This demonstrates how the hydrostatic pressure is distributed among the various resisting members of the structure according to Eq. 9. The considerable contribution of the slab-like function of the lower portion of the structure can thus be seen.

The Rio Freddo Dam tested at the ISMES Laboratory is of interest because this thin-arch dam does not abut directly onto the rock, but instead against imposing gravity buttresses, as shown in Fig. 11. The Pontesei Dam, which is also a thin-arch dam, indicated very good behavior despite the comparatively

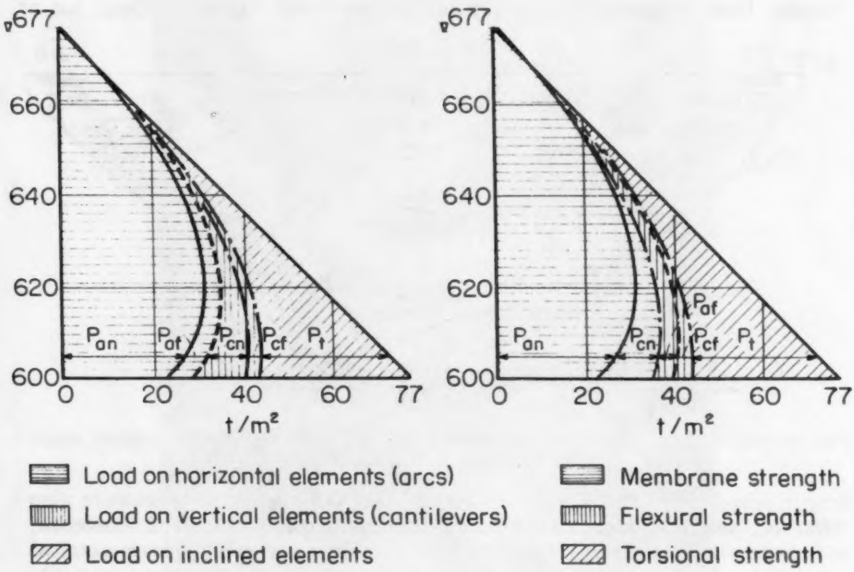

FIG. 10.—HYDROSTATIC PRESSURE DISTRIBUTION AT VAL GALLINA DAM.

high value of 850-psi stresses allowed in the design stage, and is also interesting because the same model was utilized for an experimental study of a difficult problem. It was desired to ascertain the possibility of damming a narrow gorge formed by two high-resistance rocky side walls, but whose bottom foundation material was relatively weak. It was thus necessary to investigate the ability of the structure to sustain itself by discharging sideways the actions produced by its dead load if the bottom support should fail. This test was carried out by measuring the stresses induced in the model by the dead load effect reproduced by alternate cycles of loading and unloading, after having cut a horizontal slot through the base of the model passing from the upstream to the downstream face.

In addition to the problem of double-curvature dams in broad gorges, experimental studies have been made of the inverse problem of arch dams to be built across U-shaped, high and narrow gorges having nearly parallel walls. The two main cases were: the 500 ft high Santa Giustina Dam, having a 260 ft mean span, and the more recent 870 ft high Vajont Dam, whose mean span of about the same order of magnitude as that of the former dam, has a tendency to enlarge at the upper zone. Very recently we have also tested the 330 ft high Val Noana Dam.

Tests conducted on high and narrow dams have revealed that the static behavior of the latter is quite different from that of the dams in broad valleys previously discussed. Indeed, the arch effect in them prevails over the cantilever effect already at normal loads, and in case of a sufficiently symmetrical structure, the isostatic lines do follow the arch-cantilever lattice pattern.

The final design of the Vajont Dam, which was completed to a height of 870 ft in 1957, was the object of model tests at the ISMES Laboratory and the results

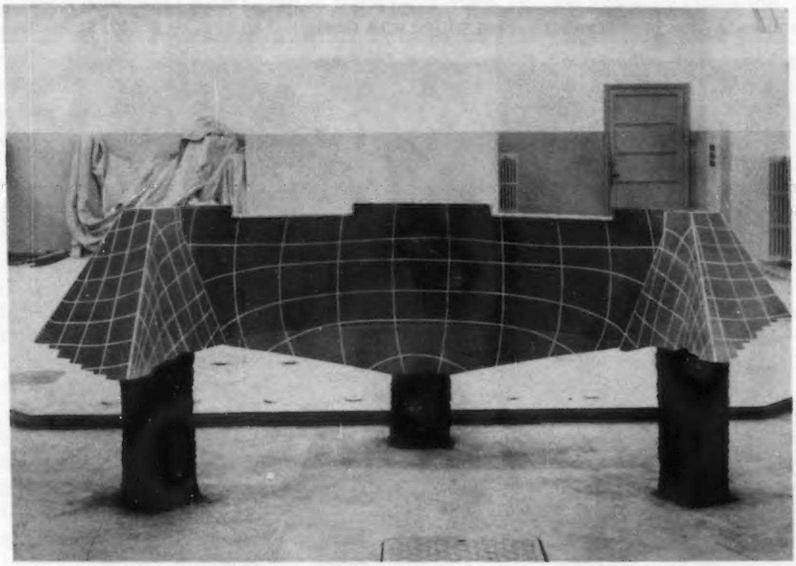

FIG. 11.—ISOSTATIC LINES ON RIO FREDDO DAM MODEL.

obtained were very satisfactory. The whole upstream face of the dam was found to be stressed to a maximum of 850 psi. There were moderate tensile stresses on the downstream face to a maximum of 70 psi to 85 psi and compressive stresses on the order of 900 psi. The overall safety factor was found by ultimate load tests to be above 7.

The Lumiei Dam is an intermediate type when compared to the types discussed above. This dam, finished in 1947, was considered exceptional in those days, not only for its 450 ft height, which made it the world's highest thin-arch dam, but because the gorge is V-shaped and has a height about equal to the crest span. Tests conducted on 1:60 scale models cast in plaster-celite mixes, were aimed at comparing the results obtained under normal elastic con-

ditions with those furnished by calculation. Later, those tests were continued up to the model's collapse, to check its margin of safety. Results of these tests have been published previously.

In concluding, the high values of the overall safety factor of 7 to 12 that are always obtained in the ultimate load tests of this kind of thin arch dam should be emphasized. The cause of this high factor of safety is to be found in the vast strength resources characteristic of arched structures, particularly doubly curved arch dams, capable of sustaining very high loads with extremely small deformations. Such resources of strength may partially be revealed by analytical methods, and only model tests can uncover them to their full extent. Ultimate collapse of these structures depends above all on the strength of the abutment rock.

DYNAMIC TESTS OF THIN ARCH DAMS

Recently, dynamic studies were conducted at the ISMES Laboratory for the Ambiesta double-curvature dam which was designed to create a storage reser-

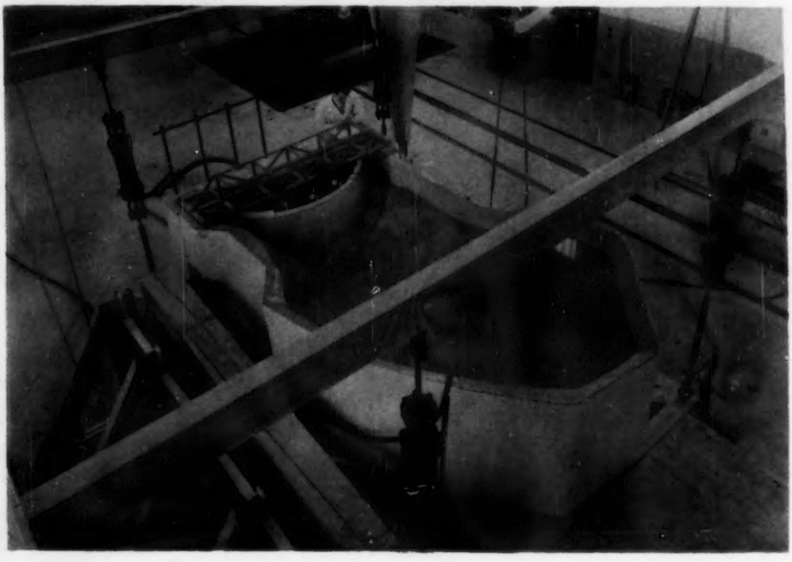

FIG. 12.—ISMES LABORATORY VIBRATING STEEL PLATFORM WITH AMBIESTA DOUBLE-CURVATURE DAM MODEL.

voir in a highly seismic region of the Friuli province. The models (Fig. 12) were cast in special plaster-litharge mixes so as to conform to the conditions of dynamic similitude, and were tested on a vibrating platform operated by a 10-ton vibrator with a maximum frequency of 25 cycles per sec. Successive models were used in order to test the various seismic effects caused by horizontal and vertical vibrations. The tests furnished useful indications about the dynamic behavior of the structure, which was found to be satisfactory on the whole.

ARCH-GRAVITY DAMS

The Pieve di Cadore Dam, completed in 1949, represents the first exhaustive investigation of the arch-gravity dam by the ISMES, and it stands out as a conspicuous example of its type. With a mean height of 180 ft, this dam has a span of 1,080 ft with a large central angle. The maximum height, at the deep gorge through which the river flows, is 340 ft. The actual arch section of the dam, which is almost symmetrical with respect to the central vertical plane, is founded in calcareous rock except for a small portion which rests on the concrete plug closing the deeper part of the gorge. The static behavior of this plug had been completely investigated by means of models. But the most important testing was conducted on the model of the entire dam, forming an enormous, curved, variable-thickness slab and presenting factors which, even though not calculable, exert an influence on the stability of the structure. These factors are the asymmetry caused by the presence of the plug, the existence of perimetral and radial joints, and the irregular outline of the abutments. The tests were carried out in 1947 and 1948 at the large double-deck tank of the ISMES on two 1:40 scale models. Considerable displacements were found at the base, particularly in the central zone, which profoundly influenced the trend of the overall deformations. The principal stresses deduced from the test results were found to be comparatively moderate, and well within the allowable stresses.

The results furnished a first criterion for judging the static efficiency of the structure. However, it seemed important to elaborate on these in order to gain deeper insight into this judgment and to compare the results of the model studies with those obtained by various methods of calculation, in order to draw conclusions of a general nature. The theoretical investigation was limited to the central portion of the dam, the remainder being so complicated by local boundary conditions as to preclude any attempt at studies except on a model. In the central portion the isostatic lines follow the traditional vertical-horizontal lattice pattern of cantilevers and arches, and the stresses there are distributed with fair regularity on both faces of the dam.

The evaluation of the distribution of the hydrostatic load among the various elements of resistance of the dam, indicated in Eq. 9, led to the conclusion that the dam was almost exclusively supported by the axial resistance of the arches and by the flexural strength of the cantilevers, as shown in Fig. 13. The contribution of the flexural strength of the arches was found to be very small, with a slight tendency to increase toward the base of the dam. The effect of the moderate second curvature was minor.

In addition to the Pieve di Cadore Dam, which was the first of many arch-gravity dams tested at the ISMES Laboratory, some others deserve mention because of the singularity of the problems they present and in order to emphasize the substantial contribution afforded by model testing to the solution of such problems. The 340 ft high and 890 ft span Forte Buso Dam, finished in the fall of 1952, was peculiar because it dammed a highly asymmetrical gorge. The Fedaia Dam, which was subsequently built as a buttress dam, was important for its exceptional span-to-height ratio of over 6 and because of the overhanging upper part.

FOUNDATION EFFECTS

The Beauregard Dam is an arch-gravity dam with a height of 433 ft, a crest length of about 1,300 ft, a thickness increasing from a minimum of 16 ft at the

top to a maximum of 147 ft at the base, a radius of curvature of 535 ft, and a central angle at the crest of 132°. The main peculiarity of this dam is the large difference in the mechanical properties of the foundation rock at the two abutments, the ratio between the moduli of elasticity of the two abutments being 1:10. A further complication arose from the presence of a pocket of loose material at the foot of the weaker abutment of such proportions as to require the construction of an important concrete subfoundation.

Once a preliminary design of the dam had been worked out, a 1:50 scale model was built which faithfully reproduced the foundation characteristics, including the pocket of weak material, and was tested in the ISMES testing tower. The results of this test permitted improvement of the design through a greater asymmetry and a reduction of volume, thus simultaneously increasing the static efficiency of the structure and reducing its cost. This model was consequently modified and subjected to a new series of tests, as shown in Fig. 14. The deflections of the downstream face indicated clearly the influence of the different

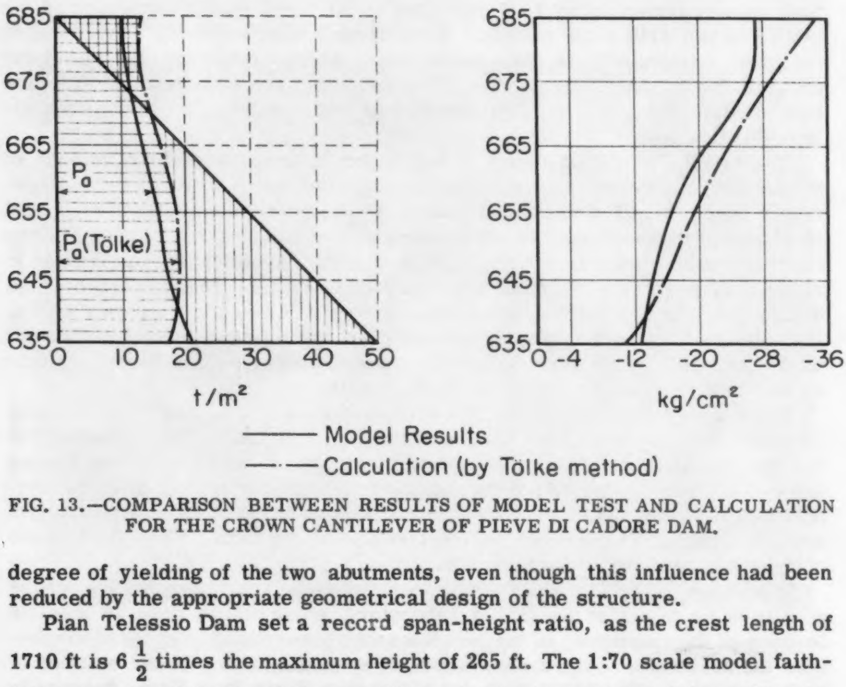

FIG. 13.—COMPARISON BETWEEN RESULTS OF MODEL TEST AND CALCULATION FOR THE CROWN CANTILEVER OF PIEVE DI CADORE DAM.

degree of yielding of the two abutments, even though this influence had been reduced by the appropriate geometrical design of the structure.

Pian Telessio Dam set a record span-height ratio, as the crest length of 1710 ft is $6\frac{1}{2}$ times the maximum height of 265 ft. The 1:70 scale model faithfully reproduced the foundation conditions, having been cast in deformable materials that presented moduli of elasticity nearly identical to those of the dam, as was demonstrated by field tests. After being subjected to the equivalent of the real dead load, the model was investigated only for the action of the hydrostatic head applied at open radial joints. Test results, both in the elastic range and under ultimate-load conditions, were found highly satisfactory.

Model investigations were carried out in 1957 of the large Reno di Lei Dam at the Italian-Swiss border with very good results. This dam exceeds, both in

size and span-to-height ratio, the Pian Telessio Dam, reaching the highest values yet attained.

CONSTRUCTION BY STAGES

Model tests have showed the behavior of some large arch-gravity dams whose construction was to be carried out in successive stages.

Frera Dam, which was under construction in 1958, will reach a final height of 460 ft and a span of nearly 1,150 ft in three successive stages. This dam has been the object of numerous model tests, which were discussed in papers presented at the 6th Congress on Large Dams. The new Cancano Dam, which was designed by the writer in cooperation with Felice Contessini, is to be erected in two successive stages. The first-stage construction, which has already been completed, reaches a height of 450 ft and a span of 1,080 ft. This will

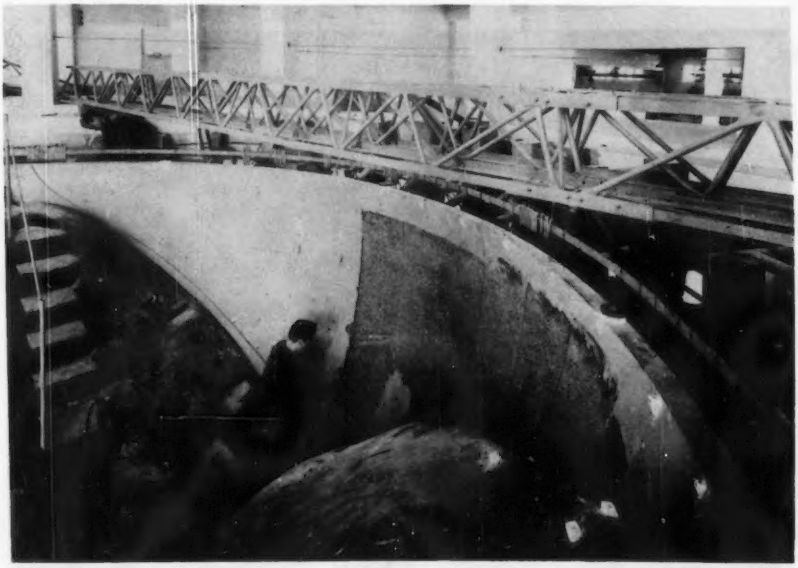

FIG. 14.—MODIFICATION OF DOWNSTREAM FACE OF BEAUREGARD DAM MODEL.

later be incorporated into the second stage concrete, and the dam's ultimate height will be 580 ft with an arch span at the crest of over 1,300 ft. The foundation rock is considerably anisotropic due to the inclination of bedding planes of the rock with respect to the vertical axis of the valley, so that the flow of the stresses from the dam to the rock is perpendicular to the rocky strata on one side, and nearly parallel to the strata on the other side. The early 1:50 scale model of the first-stage construction, shown in Fig. 15, faithfully reproduced the anisotropic characteristics of the rock by interposing, between layers of the foundation material, rubber sheets conveniently perforated to permit their lateral expansion. The test showed, particularly by the trend of the isostatic lines, how a rational dimensioning of the structure may lead to a con-

siderable reduction of the feared effects of asymmetry caused by the anisotropic foundation. Ultimate-load tests proved the structure's high factor of safety in this case, too.

Model tests revealed that arch dams built in successive layers, or "onion-like" dams, furnish a very favorable static behavior, as the cantilevers of different stages tend to unite closely like staves of a barrel pressed from the outside, finding a valuable support in the final stage's crest arch, which is thus usefully compressed. Such characteristic behavior is not to be found in straight gravity dams.

CONCLUSIONS

The first part of the paper deals with the general criteria for the design of modern large arch dams and describes briefly the development of the design and investigation of such dams in Italy.

FIG. 15.—NEW CANCANO DAM: DOWNSTREAM VIEW OF MODEL AT ISMES TESTING TOWER. ISOSTATIC LINES, OBTAINED BY TESTING, ARE SHOWN ON THE SURFACE OF THE MODEL.

The shape of the arch dam must conform to the conditions imposed by the morphological characteristics of the dam site. A rational solution to the problem can be found using the general calculation criteria set forth in the second part of the paper, taking into consideration the aesthetic and economic requirements of the dam, and utilizing the contribution afforded by the second curvature. In the third and last section, the structural analysis methods used in model testing are briefly outlined, and then illustrated with results obtained in some of the most remarkable investigations carried out at the ISMES in Bergamo in connection with the most recently built Italian dams.

These model investigations have made it possible to obtain reasonable economy in dam design. They have, furthermore, permitted the evaluation of the overall safety factor of structures designed with great asymmetry, heterogeneous and irregular foundations, and perimetral joints and openings inside the dam as well as other peculiarities which could be subjected to study only through model testing.

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CONTENTS

DISCUSSION

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ANALYSIS OF A TWO WAY TRUSS SYSTEM[a]

Closure by John A. Sbarounis and Michael P. Gaus

JOHN A. SBAROUNIS,[1] M. ASCE and MICHAEL P. GAUS,[2] A. M. ASCE.—As Mr. Bravo has pointed out, the analysis of a grid of orthogonal members can be carried out by applying the theorem of three moments to establish moment-deflection and "compatibility" relationships. This method could be very useful if it is not necessary to consider any torsional effects arising from differential rotation of the ends of the beams forming the grid. In the case of the dining-hall truss system, the torsional rigidity of the trusses was found to be very small and was therefore eliminated as a factor in the computations. Such an elimination of torsional effects may not always be possible if the grid is composed of members having a large torsional rigidity, and a "twisting" term would then be carried in the difference equations.

The method of supporting the dining-hall truss system created another difficulty in the analysis. It was found in applying the difference-equation method that the between-truss support could be included in the equations with no particular difficulty. This was not the case for the consistent-deflection analysis and would also be a problem in using the three-moment approach.

The comments and derivations given by Mr. Schjodt are very interesting in that he uses a slightly different boundary condition in developing the difference equations. A comparison in the paper of the chord stresses computed by two methods, and given in Fig. 24, should not be expected to agree around the supports for several reasons, which perhaps were not properly emphasized. These large percentage differences are not attributable to inherent differences in each method of analysis or to "inaccuracies" of the difference equations, but are principally due to the fact that, in order to save time, the initial analysis by consistent deflections did not use the actual type of support, and it was simplified by eliminating several trusses and introducing their stiffness and loads into adjoining trusses. Also a number of changes were made in member sizes between the two analyses. The comparison was primarily given to show that no radical changes took place in the stresses when member sizes were changed and a more detailed analysis of a slightly different type was used.

In application of difference equations it is usually the boundary conditions that cause difficulty. The difference equations presented in the paper were developed from the equations indicated using Taylor's series expansions. However, the same equations can be developed by using a physical analogy which breaks an elastic structure into a series of rigid bars connected by elastic

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[a] February, 1959, by John A. Sbarounis and Michael P. Gaus.

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[2] Ass't. Prof., Pennsylvania State Univ., University Park, Pa.

joints. Any distributed loads are converted to statically equivalent loads acting at the joints, and the elastic properties of the member on half a grid spacing to each side of the joint are concentrated at the joint. The development of difference equations can then be carried out by considering the equilibrium of the analogous structure. This approach is often used to treat "bad" boundary conditions encountered in plate or orthotropic plate problems with the grid simply being the case of a perforated orthotropic plate.

SELF-CHECKING GENERAL ANALYSIS OF RIGID FRAMES WITH SWAY[a]

Closure by F. G. Keller

F. G. KELLER.[1]—When first prepared, this paper carried the title "A General Analysis of Complex Plane Rigid Frames using Moment Distribution Combined with Graphic Statics." To bring this title down to a more reasonable length the reference to moment distribution and graphic statics was deleted. However, in the Synopsis, the process is described clearly as being a combination of moment distribution and graphic statics.

It has never been claimed that this method would always be more expedient than other available methods. On the contrary, the final paragraph of Section A admits that the problem of a simple portal frame can "obviously be solved quicker by other means," but emphasis was placed on the suitability of this method to solve reliably not only complex structures but also simple frames. It is held that in spite of the admittedly lengthy computations (when the checking of moments, axial forces, and shears is included) this process might be used reliably by the less highly trained engineer.

Evidently Mr. Polivka prefers his own method which is based on the fixed-point theory. It is noted, however, that this procedure necessitates estimating some coefficients of resistance and the full extent of all calculations necessary is not shown by the discusser when dealing with the complex frame solved in the subject paper.

It is agreed that it would represent a great advantage if the bulk of his calculation could be re-used when analysing for other loading conditions.

In this connection it might be pointed out that the writer's method also contains "stock computations" which can be re-used in case of changed loading conditions; these are sway-moment distribution, determination of joint forces, and the corresponding Maxwell diagrams. Thus for changed loading conditions of the frame as shown in Fig. B. 1, only one more moment distribution followed by one Maxwell diagram will supply the new coefficients r_{10}, r_{20}, etc., in Eq. 5, which will have to be solved for the K-factors. The ensuing final moment and force computation can again be checked by a final Maxwell diagram if confirmation of the results is desired.

A final Maxwell diagram may be drawn also in cases where frames had been solved by other methods of analysis. Though such a check in itself is not conclusive, it would be in most cases expose an incorrect set of final moments.

Similarly, an advantage of the writer's method is seen in the fact that very little additional work is required to investigate the set of bar stresses, if a support is removed or an extra support introduced.

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[a] February, 1959, by F. G. Keller.

[1] Lecturer N.S.W. Univ. of Technology, Sydney, Australia.

Referring to Fig. B 1, if the influence of a horizontal restraint at point E were to be investigated, a great part of the previous computations could be used without alterations. For "no sway" the imaginary restraint at D was 16, while for "Sway 1" it was 74.

Therefore only one equation would be required: $16 - 74\,K = 0$, from which $K = 8/37$.

All moments from Sway 1, as obtained previously, would have to be multiplied by 8/37 and then added to the corresponding "no sway" moments, as found previously. This would give a set of final moments for the new fixation version. A final Maxwell diagram, which must comply with the requirement that the horizontal restraint at D = 0, will yield all axial forces and the horizontal reactions at E, exposing the influence of such a variation of the boundary conditions.

With regard to the requisite for conclusive checking of the results, the writer wishes to qualify an incomplete statement in the paper. The requisite for the conclusive checking of the set of moments found is the closing of the final Maxwell diagram, provided requirements 1 to 4 as stated previously were satisfied.

The same requisite, however, extends to all axial and shear forces only, if the structure with its loading and reactions satisfies the moment equations for equilibrium. This latter requirement is, of course, a routine check, which most engineers would do in any case. Without it, however, there could still exist the possibility of an error in the establishment of the final joint forces; this will be shown with reference to Problem A. Suppose an error had occurred in the division of the sum of beam end moments by the length of beam: $\frac{1}{20}\left(-10\frac{5}{7} - 10\frac{5}{7}\right) = -\frac{15}{14}$; the Maxwell diagram would still close, but only for the correct value of $\frac{15}{14}$ would the moment equation (about D) be satisfied:

$$5 \times 10 = \frac{15}{14} \times 20 + 14\frac{2}{7} + 14\frac{2}{7}$$

Referring to Mr. Polivka's statement as to the use of Mohr checks, when dealing with symmetrical structures and/or elastic symmetry, the writer cannot concur to the views expressed. In the general case of a frame consisting of n cells, 3 n Mohr equations will be available. They can be written for each cell as:

$$\sum\left(\frac{M}{E\,I}\;\Delta l\right) = 0;\quad \sum\left(\frac{M}{E\,I}\;\Delta l\;\; y\right) = 0;\quad \sum\left(\frac{M}{E\,I}\cdot\Delta l\;\cdot\; x\right) = 0,$$

expressing the fact that relative rotation, relative horizontal and vertical shifts respectively between terminals of each cell are zero. Due to symmetry, some of the redundant end moments become numerically identical, thus reducing the number of unknown moments, while some of the Mohr equations become meaningless.

In the case of a symmetrical single-cell structure with loading as in example A, $M_A = M_D$ and $M_B = M_C$; consequently only two of the four values for moments have to be confirmed. The third requirement above, when referred to an axis along one of the columns, yields one relationship between the two remaining unknown moments. A further relation between these is obtained by statics. The computations are

$$\frac{-M_D \times 10}{2} \times 20 + \frac{M_C \times 10}{2} \times 20 + \frac{1}{2} \cdot \frac{M_C \times 20}{4} \times \frac{20}{3} = 0$$

$$\text{or } 4\,M_C = 3\,M_D \qquad (1)$$

This checks with the writer's results. The shear equation yields

$$2 \times \frac{M_C + M_D}{10} = 5 \qquad \text{or} \quad M_C + M_D = 25 \qquad (2)$$

From Eqs. 1 and 2

$$M_D = +14\,\frac{2}{7}$$

$$M_C = +10\,\frac{5}{7}$$

As a matter of fact, these "Mohr check equations"—as an alternative—can always be used to solve a statically indeterminate frame by writing as many of them, that when supplemented by the available relations of statics, they will yield a number of simultaneous equations equal to the number of different end-moments acting in the bars of the structure. This method has been named by the writer "The Direct Method." Moments are introduced at the ends of bars with algebraic signs depending on the tendency to turn to the right or left, when tracking along a cell, allowing for an assumed deformation pattern of the frame.

In the case of a symmetrical two-cell portal on three unyielding simple supports at the same level, all members having constant E I, with a single horizontal load acting along the beam, there will exist three different unknown moment values at the ends of members. Due to symmetry, this number of unknown moments is reduced by two. The frame is three-fold statically indeterminate.

For a complete confirmation of end-moments, as many independent Mohr checks are required as the structure is statically indeterminate, but due to symmetry the number is reduced by two. Thus in the foregoing case only one Mohr check is required and this is a statement for the absence of a horizontal shift between the external and central supports.

The supplementary relations of statics are the shear equation and an equation expressing rotational equilibrium at the top of the central column. The three unknown moments can therefore be obtained by the solution of three simultaneous equations.

The further available Mohr equations are not required for the establishment of the moment diagram. It should be noted, however, that the vertical-shift equations would not be complete without including the rotations at the supports which represent further elastic weights. If desired, these rotations could be determined by the requirement of equilibrium of the elastic weights, considering each cell separately and taking moments about the column axes.

It is easily seen that the cell comprising both outer columns cannot be used for this purpose in such a symmetrical arrangement, as all three Mohr equations here would become meaningless.

From the foregoing it is evident that a set of moments, obtained by whatever method of analysis, can be checked readily by the Mohr method, and this includes cases of structural or elastic symmetry.

[illegible]

SHORT FLEXIBLE SUSPENSION BRIDGES FOR HEAVY TRUCKS[a]

Closure by S. O. Asplund

S. O. ASPLUND,[1] F. ASCE.—Mr. Eremin questions an obscure statement in the paper that the movements of these bridges "need attention." The phrase "in design" was understood by the writer. That was also intended to be a major point in the paper.

Mr. Eremin's question, however, has real significance because special attention to movements surely is also needed during the life of the bridges. Some of the bridges have been crossed daily by thousands of loaded 40-ton trucks for years. Thus all wear or other defects must be discovered, closely watched and corrected whenever necessary. Also, in this way, the bridges certainly need and have received attention, and it has cost $5 for each day of heavy traffic.

The rough traffic has, in fact, resulted in wear of incorrectly designed or fabricated movable joints and other details. Some faults in fabrications have been discovered by the daily inspections. Progressive wear has been observed mainly in the hanger connections, near mid-span, where the angular movements are considerable. Looped hanger connections have been replaced with bolted joints, as described in the paper. For later bridges, the importance of not making the mid-span hangers too short was learned. As it should, the cost for repairs has decreased for every new bridge designed. It should now be much less than $1,000 per yr of traffic. For example, the cost of materials, wages, and insurance was for the Harrsele Bridge $570, $470, and $530 during the years 1954, 1955, and 1956, after which it was dismantled. About 25% of these costs was for maintenance of the splices in the longitudinal girders. This cost has been practically eliminated in the newer bridges with high-tension bolt splices.

Mr. Eremin seems to question whether design measures for reducing the deflections should not be taken. He illustrates in Fig. 30 possible arrangements of split main cables. The arrangement in Fig. 30(a), where each cable trajectory carries one half of the span, resembles somewhat the system Ordish-Léfeuvre.[2,3,4,5]

Thoughts of reducing the deflections seem to be as old as suspension bridges themselves. Many solutions can be found in literature. The idea is in itself right; the deflections can be reduced, perhaps by as much as one quarter. That

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[a] April, 1959, by S. O. Asplund.

[1] Tekn. Dr., Prof. of Structural Mechanics, Chalmers Univ. of Technology, Gothenburg, Sweden.

[2] Zeitschr. Arch. u. Ing. Ver. Hannover, Vol. 30, 1884, p. 56.

[3] Claxton Fiedler's system, Engineering, Vol. 19, 1885, p. 372.

[4] Max am Ende, see Proc. Inst. of Civ. Engrg., Vol. 137, London, 1899, p. 306.

[5] "Improved type of suspension bridges," by Carl Forssell, Teknisk Tidskr., Stockholm, 1917.

is a considerable reduction, bearing in mind that about half of the deflections are very often due to the elastic elongations of the cable, which cannot very well be reduced. Another trend in design is, however, that the deflections in themselves are actually harmless and therefore negligible (Civil Engineering 1953, p. 480). This line seems victorious. After complicated arrangements have, at times, been tried, suspension-bridge designers, for clear-cut calculation, erection, and other reasons, mostly have returned to the simple parabolic cable with vertical hangers and a stiffened roadway. In 1947 the writer built a small bridge with cables similar to those shown in Fig. 30. It had three cable trajectories, each carrying one third of the main span. This design considerably reduced the deflections. Still it is questionable whether the complications in erection, or in many details, for instance of joints like B in Fig. 30(a), did not offset the gains in increased stiffness.

The writer made some model measurements[6] on a short-span suspension bridge with one undivided cable and various arrangements of inclined hangers. He could not measure a larger reduction in deflections than about 15%. This gain was considered too small to motivate the use of inclined hangers instead of vertical hangers for stiffness.

Mr. Eremin should be thanked for having posed questions needed for explaining unclear and missing points.

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[6] Tekn. Tidskr. 1940, p. V 23.

STABILITY CONSIDERATIONS IN THE DESIGN OF STEEL COLUMNS[a]

Discussions by A. A. Eremin and Theodore V. Galambos

A. A. EREMIN,[1] M. ASCE.—In this paper the author has shown an interesting bending-torsion method of analysis of stresses in a column.

The column loading considered by the author consists of the end boundary conditions, expressed by the bending moments and direct forces. In practice, the column loading may also consist of the torsion moments at the ends, which were not considered by Mr. Massenet. The end torsion restraint is, however, necessary to accommodate the resistance to the torsional stresses developed in the buckling of a column.

The variation of the bending stresses in the column has been expressed by the approximate trapezoidal formula, Eq. 17. It is interesting to note that Eq. 17 gives a slight reduction in the maximum moments, with the ratio of moments varied from zero $r = +1$. The same equation, when applied to the end bending moments with ratio $r = -1$ reduction of stresses, is larger and obviously affects continuity of critical stresses.

The author has made some practical suggestions concerning the application of the bending-torsion formulas. It would be of interest to determine the resulting coefficient of safety.

The author deserves high commendation for his clear interpretation of the method and for the numerous illustrating tests shown in his Fig. 23. From the total 91 tests the stresses in 83 tests were located close to the vicinity of the straight line in Fig. 23.

THEODORE V. GALAMBOS,[2] A. M. ASCE.—Excellent summary of the stability problems encountered in the design of steel columns has been presented.

Extensive reference is made by the author to a paper by R. L. Ketter and the writer which was published in April, 1959.[3]

The interaction curves[3] have been reduced to analytical expressions by curve-fitting, and these formulas are the basis of column design as proposed in the recently published AISC PLASTIC DESIGN MANUAL.[4]

The following items of discussion are presented in order to supplement the author's treatise on the stability of steel columns:

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[a] September, 1959, by Charles Massonet.

[1] Assoc. Bridge Engr., Calif. State Highways, Sacramento, Calif.

[2] Research Ass't. Prof., Fritz Engrg. Lab., Lehigh Univ., Bethlehem, Pa.

[3] "Columns Under Combined Bending and Thrust," by T. V. Galambos and R. L. Ketter, Proceedings of the American Society of Civil Engineers, Vol. 85, EM 2, April, 1959, p. 1.

[4] Amer. Inst. of Steel Constr. N. Y., 1959.

1. The author correctly concludes that the hypothetical maximum stress due to the "secant" type solution of the problem of plane bending under axial load is the yield stress (Eq. 3). However, it is not pointed out that yielding of the column will commence before the elastic-theory yield stress is reached in the most stressed fiber of the column. This is because of the presence of residual stresses, which can attain a magnitude of about 50% of the yield stress for rolled wide-flange columns. This negative effect counteracts part of the beneficial effects discussed by the author (for example, reserve of strength due to plasticity), and in certain cases the "initial yield" or "secant" theory may lead to unconservative designs. This was pointed out by Ketter, Kaminsky and Beedle.[5] In this paper, a curve for ultimate strength, including the effect of a

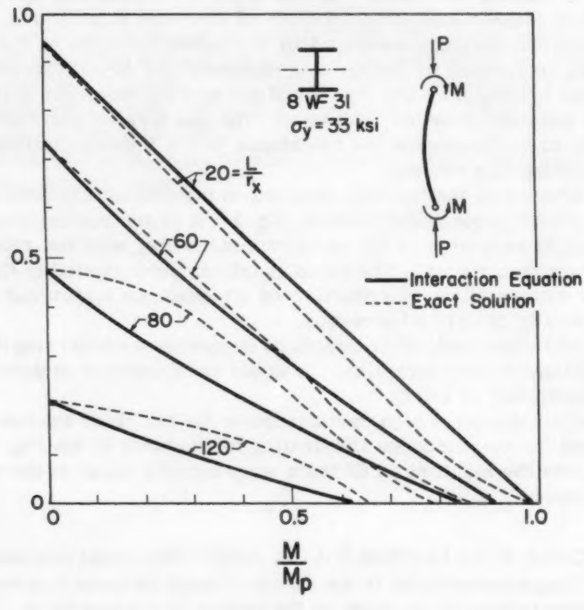

FIG. 1

compressive residual stress of 0.3 y, is compared with a curve for an "initial yield" solution. The latter is seen to be unconservative in the region where the axial load is relatively high and the bending moments are small.

2. The writer has recently completed a solution for the inelastic lateral-torsional buckling strength of rolled wide-flange columns subjected to axial forces and equal end bending moments causing single curvature deformation of the column axis ($M_{equiv} = M$ in the author's Eq. 40).[6] The influence of residual stresses is also included in the calculation. The procedure of solution is

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5 "Plastic Deformation of Wide-Flange Beam-Columns," by R. L. Ketter, E. L. Kaminsky, and L. S. Beedle, ASCE Transactions, 1955, p. 120, Fig. 36.

6 "Inelastic Lateral-Torsional Buckling of Eccentrically Loaded Wide-Flange Columns," by T. V. Galambos, Ph. D. Dissertation, Lehigh Univ., 1959.

an Eigenvalue process, where the reduction of the various stiffnesses governing lateral-torsional buckling due to yielding is taken into account. No further description of the method is included herein, as it is quite complicated. A comparison of interaction curves obtained by this method and by the author's interaction formula (Eq. 40) for lateral-torsional buckling is shown in Figs. D1 and D2. The solid lines refer to the interaction equation (author's Eq. 40, where P_o is taken as the weak-axis buckling load of a pin-ended column, and M_o is the critical moment under pure bending, as computed by Eqs. 45), and the dashed lines represent the discusser's "exact" solution. Fig. D1 is for the 8WF31 shape, and Fig. D2 for the 14WF 142 shape. It can be seen from these figures that the interaction equation is, in general, conservative, except in some limited regions where it furnishes somewhat unconservative answers.

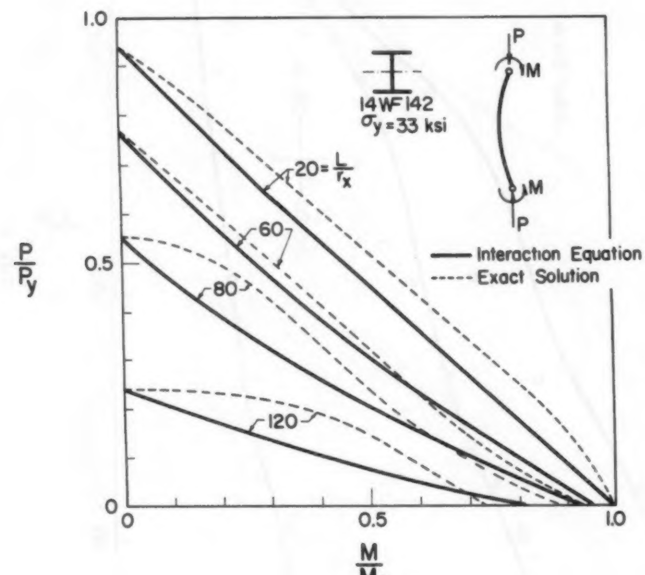
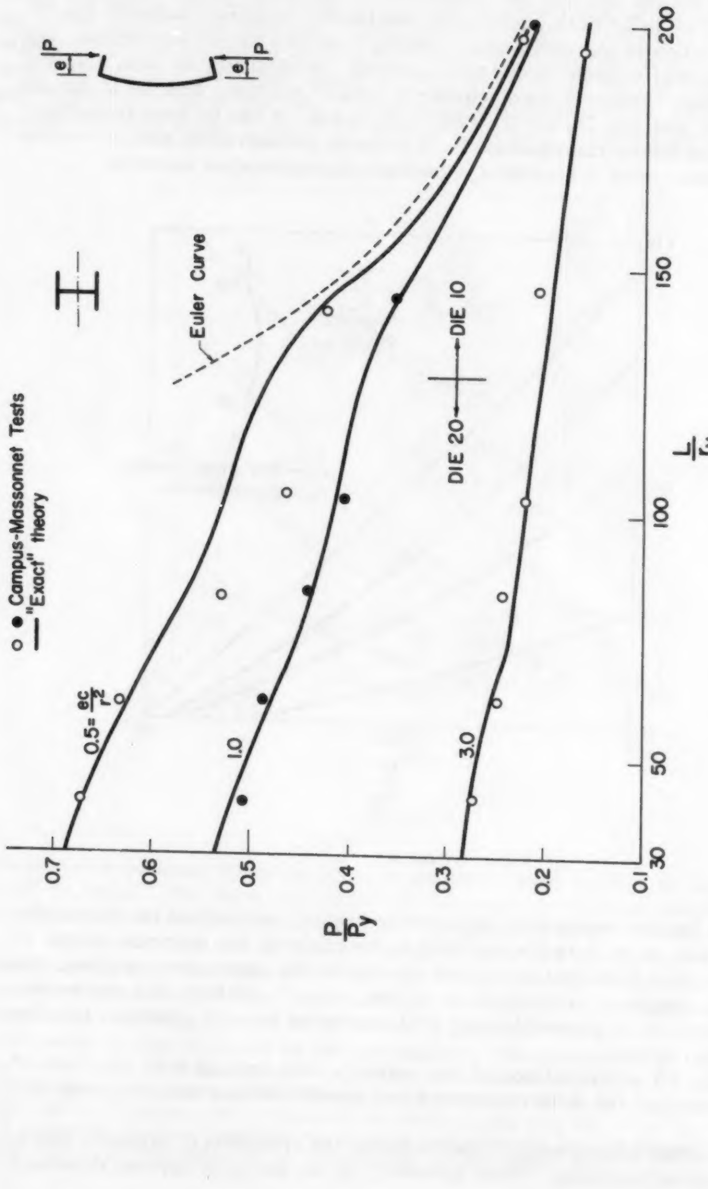


FIG. 2

Although the correspondence between the "exact" method and the interaction equation is not as good as for buckling in the plane of the moments (Figs. 21 and 22), it is still sufficient to warrant the use of the interaction equation. Because of the extreme laboriousness of the "exact" method, the interaction equation proposed by the author may well represent the only practical solution for the design office.

3. In Fig. D3 a comparison of the author's test results with the "exact" method shows that the writer's method can predict column behavior with fair accuracy.

4. The author places much importance on the reduction of strength due to lateral-torsional buckling. This, however, is not the most serious drawback



○ ● Campus-Massonnet Tests
—— "Exact" theory
Euler Curve
DIE 20
DIE 10
0.5= ec/r²
1.0
3.0
e
P
0.1
0.2
0.3
0.4
0.5
0.6
0.7
P/Py
30
50
100
150
200
L/ry

FIG. 3

of this type of failure. In fact, maximum strength may often be quite well predicted by neglecting the effect of lateral-torsional buckling,[7] except for columns pinned in both directions at the ends and subjected to equal or nearly equal end moments. The most serious consequence of lateral-torsional buckling is that it may reduce the rotation capacity of a column. Such an effect is precluded through the use of appropriate lateral bracing.[8]

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[7] Columns Under Combined Bending and Thrust, by T. V. Galambos and R. L. Ketter, Proceedings of the American Society of Civil Engineers, Vol. 85, EM 2, April, 1959, see Figs. 12 through 15 of Reference 15.

[8] "ASCE-WRC Commentary on Plastic Design - Compression Members, ASCE Proc. Paper, 86, EM 1, January, 1960.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 2270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural Division during 1959.

VOLUME 85 (1959)

MARCH: 1960(HY3), 1961(HY3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968(SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977(WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986 (IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

APRIL: 1990(EM2), 1991(EM2), 1992(EM2), 1993(HW2), 1994(HY4), 1995(HY4), 1996(HY4), 1997(HY4), 1998 (SM2), 1999(SM2), 2000(SM2), 2001(SM2), 2002(ST4), 2003(ST4), 2004(ST4), 2005(ST4), 2006(PO2), 2007 (HW2)[c], 2008(EM2)[c], 2009(ST4)[c], 2010(SM2)[c], 2011(SM2)[c], 2012(HY4)[c], 2013(PO2)[c].

MAY: 2014(AT2), 2015(AT2), 2016(AT2), 2017(HY5), 2018(HY5), 2019(HY5), 2020(HY5), 2021(HY5), 2022(HY5), 2023(PL2), 2024(PL2), 2025(PL2), 2026(PP1), 2027(PP1), 2028(PP1), 2029(PP1), 2030(SA3), 2031(SA3), 2032(SA3), 2033(SA3), 2034(ST5), 2035(ST5), 2036(ST5), 2037(ST5), 2038(PL2), 2039(PL2), 2040(AT2)[c]. 2041(PL2)[c], 2042(PP1)[c], 2043(ST5)[c], 2044(SA3)[c], 2045(HY5)[c], 2046(PP1), 2047(PP1).

JUNE: 2048(CP1), 2049(CP1), 2050(CP1), 2051(CP1), 2052(CP1), 2053(CP1), 2054(CP1), 2055(CP1), 2056 (HY6), 2057(HY6), 2058(HY6), 2059(IR2), 2060(IR2), 2061(PO3), 2062(SM3), 2063(SM3), 2064(SM3), 2065 (ST6), 2066(WW2), 2067(WW2), 2068(WW2), 2069(WW2), 2070(WW2), 2071(WW2), 2072(CP1)[c], 2073(IR2)[c], 2074(PO3)[c], 2075(ST6)[c], 2076(HY6)[c], 2077(SM3)[c], 2078(WW2)[c].

JULY: 2079(HY7), 2080(HY7), 2081(HY7), 2082(HY7), 2083(HY7), 2084(HY7), 2085(HY7), 2086(SA4), 2087 (SA4), 2088(SA4), 2089(SA4), 2090(SA4), 2091(EM3), 2092(EM3), 2093(EM3), 2094(EM3), 2095(EM3), 2096 (EM3), 2097(HY7)[c], 2098(SA4)[c], 2099(EM3)[c], 2100(AT3), 2101(AT3), 2102(AT3), 2103(AT3), 2104(AT3), 2105(AT3), 2106(AT3), 2107(AT3), 2108(AT3), 2109(AT3), 2110(AT3), 2111(AT3), 2112(AT3), 2113(AT3), 2114(AT3), 2115(AT3), 2116(AT3), 2117(AT3), 2118(AT3), 2119(AT3), 2120(AT3), 2121(AT3), 2122(AT3), 2123(AT3), 2124(AT3), 2125(AT3).

AUGUST: 2126(HY8), 2127(HY8), 2128(HY8), 2129(HY8), 2130(PO4), 2131(PO4), 2132(PO4), 2133(PO4), 2134 (SM4), 2135(SM4), 2136(SM4), 2137(SM4), 2138(HY8)[c], 2139(PO4)[c] 2140(SM4)[c].

SEPTEMBER: 2141(CO2), 2142(CO2), 2143(CO2), 2144(HW3), 2145(HW3), 2146(HW3), 2147(HY9), 2148(HY9), 2149(HY9), 2150(HY9), 2151(IR3), 2152(ST7)[c], 2153(IR3), 2154(IR3), 2155(IR3), 2156(IR3), 2157(IR3), 2158 (IR3), 2159(IR3), 2160(IR3), 2161(SA5), 2162(SA5), 2163(ST7), 2164(ST7), 2165(SU1), 2166(SU1), 2167(WW3), 2168(WW3), 2169(WW3), 2170(WW3), 2171(WW3), 2172(WW3), 2173(WW3), 2174(WW3), 2175(WW3), 2176 (WW3), 2177(WW3), 2178(CO2)[c], 2179(IR3)[c], 2180(HW3)[c], 2181(SA5)[c], 2182(HY9)[c], 2183(SU1)[c], 2184 (WW3)[c], 2185(PP2)[c], 2186(ST7)[c], 2187(PP2), 2188(PP2).

OCTOBER: 2189(AT4), 2190(AT4), 2191(AT4), 2192(AT4), 2193(AT4), 2194(EM4), 2195(EM4), 2196(EM4), 2197(EM4), 2198(EM4), 2199(EM4), 2200(HY10), 2201(HY10), 2202(HY10), 2203(PL3), 2204(PL3), 2205 (PL3), 2206(PO5), 2207(PO5), 2208(PO5), 2209(PO5), 2210(SM5), 2211(SM5), 2212(SM5), 2213(SM5), 2214 (SM5), 2215(SM5), 2216(SM5), 2217(SM5), 2218(ST8), 2219(ST8), 2220(EM4), 2221(ST8), 2222(ST8), 2223 (ST8), 2224(HY10), 2225(HY10), 2226(PO5), 2227(PO5), 2228(PO5), 2229(ST8), 2230(EM4), 2231(EM4), 2232(AT4)[c], 2233(PL3)[c], 2234(EM4)[c], 2235(HY10)[c], 2236(SM5)[c], 2237(ST8)[c], 2238(PO5)[c], 2239(ST8), 2240 (PL3).

NOVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248 (SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9)[c], 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)[c], 2270(ST9).

DECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278 (HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287 (PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296 (SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322 (ST10), 2323(HW4)[c], 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330 (CP2).

VOLUME 86 (1960)

JANUARY: 2331(EM1), 2332(EM1), 2333(EM1), 2334(EM1), 2335(HY1), 2336(HY1), 2337(EM1), 2338(EM1), 2339(HY1), 2340(HY1), 2341(SA1), 2342(EM1), 2343(SA1), 2344(ST1), 2345(ST1), 2346(ST1), 2347(ST1), 2348(EM1)[c], 2349(HY1)[c], 2350(ST1), 2351(ST1), 2352(SA1)[c], 2353(ST1)[c], 2354(ST1).

FEBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(SU1), 2366(HY2), 2367(SU1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(CO1)[c], 2378(SU1), 2379(SU1), 2380(SU1), 2381(HY2)[c], 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)[c], 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)[c], 2391(SM1)[c], 2392(PO1)[c].

MARCH: 2393(IR1), 2394(IR1), 2395(IR1), 2396(IR1), 2397(IR1), 2398(IR1), 2399(IR1), 2400(IR1), 2401(IR1), 2402(IR1), 2403(IR1), 2404(IR1), 2405(IR1), 2406(IR1), 2407(SA2), 2408(SA2), 2409(HY3), 2410(ST3), 2411 (SA2), 2412(HW1), 2413(WW1), 2414(WW1), 2415(HY3), 2416(HW1), 2417(HW3), 2418(HW1)[c], 2419(WW1)[c], 2420(WW1), 2421(WW1), 2422(WW1), 2423(WW1), 2424(SA2), 2425(SA2)[c], 2426(HY3)[c], 2427(ST3)[c].

c. Discussion of several papers, grouped by divisions.